

# CP observables with spin-spin correlations in chargino production

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## Abstract

We study the CP-violating terms of the spin-spin correlations in chargino production  $e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ , and their subsequent two-body decays into sneutrinos plus leptons. We propose novel CP-sensitive observables with the help of T-odd products of the spin-spin terms. These terms depend on the polarizations of both charginos, with one polarization perpendicular to the production plane. We identify two classes of CP-sensitive observables; one requires the reconstruction of the production plane, the other not. Our framework is the Minimal Supersymmetric Standard Model with complex parameters.

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## 1 Introduction

Supersymmetric (SUSY) extensions of the Standard Model (SM), like the Minimal Supersymmetric Standard Model (MSSM) [1], give rise to new sources of CP violation [2]. From a mathematical point of view, this means that in the SUSY Lagrangian complex parameters enter whose phases cannot be removed by redefining the fields. The presence of CP phases can drastically alter the phenomenology of the underlying model (for a recent review, see [3]). For instance, contributions of SUSY

CP phases to the electric dipole moments (EDM) of electron, neutron, and that of the atoms  $^{199}\text{Hg}$  and  $^{205}\text{Tl}$  can be close or beyond the present experimental upper bounds [4,5], and thus in turn constrain the size of these CP phases [4,5]. These constraints, however, are strongly model dependent, see e.g. [5]. Thus measurements of CP observables outside the EDM sector are necessary to independently determine or constrain the CP phases. Furthermore, non-vanishing phases can significantly change masses, cross sections and decay branching ratios of SUSY particles, compared to the real case, see e.g. [6,7]. Hence, in determining the underlying model parameters, the effect of their CP phases has to be taken into account. The phases could be measured once supersymmetric particles are accessible at future colliders. A genuine signal for CP violation would be the measurement of non-vanishing CP-sensitive observables.

In this paper, we propose CP-sensitive observables in chargino production

$$e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp, \quad (1)$$

within the MSSM. For the processes  $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-, \tilde{\chi}_2^+ \tilde{\chi}_2^-$  the CP-sensitive terms in the amplitude squared vanish at tree level since all coupling factors are real [8,9]. The chargino mass matrix, in the weak basis, is given by

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin\beta \\ \sqrt{2}m_W \cos\beta & \mu \end{pmatrix}, \quad (2)$$

with  $M_2$  the SU(2) gaugino mass parameter, and  $\tan\beta$  the ratio of the vacuum expectation values of the two Higgs fields.  $M_2$  and  $\tan\beta$  can be chosen real and positive, while the higgsino mass parameter can be complex  $\mu = |\mu|e^{i\phi_\mu}$ . At tree level,  $\phi_\mu$  is the only CP-violating phase that gives rise to CP-sensitive observables in chargino production, while phases from other sectors can contribute at loop level [10,11]. For example, phases of the gaugino mass parameter  $M_1$ , and the trilinear coupling parameters  $A_t$  in the stop-sector, lead to rate differences in  $\tilde{\chi}_1^+ \tilde{\chi}_2^-$  production and that of the charge conjugated pair  $\tilde{\chi}_1^- \tilde{\chi}_2^+$  at the percent level [10]. For chargino decays, rate asymmetries of the partial chargino decay widths can exceed 10%, mainly due to the phases of  $M_1$  and  $A_{t,b}$  [12] (see also [13]).

Another class of promising CP-sensitive observables are based on so-called *T-odd correlations* (or *T-odd products*), see e.g. [14]. They can give rise to CP-violating effects already at tree-level, and therefore suffer not from loop suppression as rate asymmetries. Previous studies of CP-sensitive observables, based on T-odd products in chargino production and decay, have been focussing on the spin correlations between production and decay of only one chargino [8,9]. The corresponding terms in the amplitude squared involve the polarization vector perpendicular to the production plane of one of the produced charginos. Such a transverse polarization component is a genuine signal of CP violation. The transverse polarization is then retrieved

from asymmetries in the azimuthal distribution of the decay products, which can be as large as 30%, even for small  $\phi_\mu$  of order  $\pi/10$  [9]. In such an analysis of the spin-correlations, the polarization, i.e. the decay, of only one chargino needs to be considered. However, if the decays of both charginos are taken into account, one can probe their spin-spin correlations [15]. These are terms in the amplitude squared that include the polarization vectors of both charginos. The angular distributions of the decay products of the two charginos are correlated to one another due to total angular momentum conservation. Spin-spin correlations in chargino production and decay have been utilized for the determination of CP-even coupling factors [15,16]. Moreover, spin-spin correlations have been used for the definition of CP-sensitive observables in the decays of third generation squarks [17,18].

In the present paper, we propose novel CP-sensitive observables with the help of T-odd products in the chargino spin-spin correlation terms. We take the decay of both charginos into account, and consider, for definiteness, their subsequent leptonic two-body decays

$$\tilde{\chi}_i^+ \rightarrow \tilde{\nu}_\ell \ell^+ , \quad \tilde{\chi}_j^- \rightarrow \tilde{\nu}_{\ell'} \ell'^- , \quad i, j = 1, 2 \ (i \neq j) , \quad \ell, \ell' = e, \mu . \quad (3)$$

By analyzing the CP-odd parts of the spin-spin terms, we find two independent T-odd products. The first one includes the momenta of the beams and those of the two decay leptons. Thus a reconstruction of the production plane is not necessary for a measurement of the corresponding CP-sensitive observables. From an experimental perspective, this seems to be advantageous compared to CP-sensitive observables in chargino production and leptonic decays, where such a reconstruction is essentially required [8,9]. The second T-odd product which we find involves the momenta of the charginos, requiring the reconstruction of the production plane. We consider two sorts of CP-sensitive observables which we obtain from the spin-spin correlations. Defining their statistical significances, we can make a quantitative comparison of their accessibility.

The paper is organized as follows. In Section 2, we present analytical formulae for the amplitude squared of chargino production and decay  $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^- \rightarrow \tilde{\nu}_\ell \ell^+ \tilde{\nu}_{\ell'} \ell'^-$ . We identify the T-odd products that are involved in the spin-spin terms of the amplitude squared in Section 3, and define the associated CP-sensitive observables in Section 4. We present numerical results in Section 5. In Section 6, we give a summary and conclusions.

## 2 Cross section

Chargino production  $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$  proceeds via  $\gamma, Z$  exchange in the  $s$ -channel, and  $\tilde{\nu}$  exchange in the  $t$ -channel, see the Feynman diagrams in Fig. 1. The  $\gamma$  exchange vanishes for non-diagonal chargino production  $e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ . For diagonal chargino

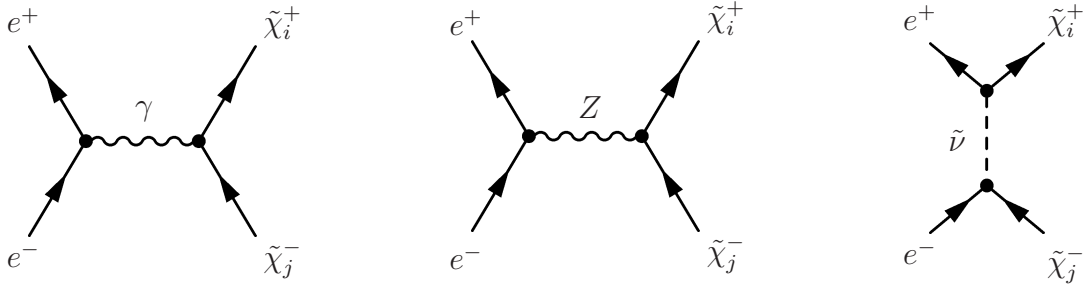


Fig. 1. Feynman diagrams for chargino production  $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$  [15].

production,  $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_i^-$ , the squared amplitude contains no CP-violating terms at tree level [8,9].

We write the differential cross section for chargino production and decay  $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^- \rightarrow \tilde{\nu}_\ell \ell^+ \tilde{\nu}_{\ell'} \ell'^-$ ,  $\ell, \ell' = e, \mu$ , generically as

$$d\sigma = \frac{1}{2s} |T|^2 d\text{Lips} , \quad (4)$$

with the center-of-mass energy  $\sqrt{s}$ , and the Lorentz invariant phase space element  $d\text{Lips}$ , which can be found in [9]. The amplitude squared  $|T|^2$  was calculated in Ref. [15] in the spin density matrix formalism<sup>1</sup>

$$|T|^2 = 4|\Delta(\tilde{\chi}_i)|^2|\Delta(\tilde{\chi}_j)|^2 \left[ P D_i D_j + \sum_{a=1}^3 \Sigma_P^a \Sigma_{D_i}^a D_j + \sum_{b=1}^3 \Sigma_P^b \Sigma_{D_j}^b D_i + \sum_{a,b=1}^3 \Sigma_P^{ab} \Sigma_{D_i}^a \Sigma_{D_j}^b \right] , \quad (5)$$

with the propagators  $\Delta(\tilde{\chi}_{i,j}) = 1/[p_{\chi_{i,j}}^2 - m_{\chi_{i,j}}^2 + im_{\chi_{i,j}} \Gamma_{\chi_{i,j}}]$  of the decaying charginos. The amplitude squared has contributions from chargino production ( $P$ ) and decay ( $D$ ). The terms  $P$  and  $D_i, D_j$  are those parts of the spin density production and decay matrices, respectively, that are independent of the polarizations of the charginos. The contributions  $\Sigma_P^a$  and  $\Sigma_{D_i}^a$  depend on the polarization basis vectors  $s_{\chi_i^+}^a$  (for their definition see Appendix B, Eq. (B.3)) of the decaying chargino  $\tilde{\chi}_i^+$ , while  $\Sigma_P^b$  and  $\Sigma_{D_j}^b$  depend on the polarization basis vectors  $s_{\chi_j^-}^b$  of the decaying chargino  $\tilde{\chi}_j^-$ . We choose a coordinate frame such that  $a, b = 3$  denote the longitudinal polarizations,  $a, b = 1$  the transversal polarizations in the production plane, and  $a, b = 2$  the transversal polarizations perpendicular to the production plane. The quantities  $D_i, D_j, \Sigma_{D_i}^a$  and  $\Sigma_{D_j}^b$  are given in Appendix A. The full expressions for the quantities  $P, \Sigma_P^a, \Sigma_P^b$  and  $\Sigma_P^{ab}$  can be found in Ref. [15].

<sup>1</sup> For a detailed discussion of the spin density matrix formalism, we refer to Ref. [19].

The contributions to the amplitude squared which depend on the polarizations of both charginos are the spin-spin correlation terms  $\Sigma_P^{ab}$ . The T-odd parts of the spin-spin correlation terms are from pure  $Z$  exchange ( $ZZ$ ) and from  $Z$ - $\tilde{\nu}$  interference ( $Z\tilde{\nu}_e$ ), and are those which include one chargino spin vector with a component perpendicular to the production plane, i.e., those with  $ab = 12, 21, 23, 32$  [15]

$$\Sigma_P^{ab}(ZZ) = \frac{g^4}{\cos^4 \Theta_W} |\Delta(Z)|^2 (L_e^2 c_{-+} + R_e^2 c_{+-}) \text{Im}(O_{ij}^{'L} O_{ij}^{'R*}) f^{ab}, \quad (6)$$

$$\Sigma_P^{ab}(Z\tilde{\nu}_e) = -\frac{g^4}{2 \cos^2 \Theta_W} \Delta(Z) \Delta(\tilde{\nu}_e)^* L_e c_{-+} \text{Im}(V_{i1}^* V_{j1} O_{ij}^{'R}) f^{ab}, \quad (7)$$

where the left and right chiral couplings of the charginos to the  $Z$  boson are

$$O_{ij}^{'L} = -V_{i1} V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^* + \delta_{ij} \sin^2 \Theta_W, \quad (8)$$

$$O_{ij}^{'R} = -U_{i1}^* U_{j1} - \frac{1}{2} U_{i2}^* U_{j2} + \delta_{ij} \sin^2 \Theta_W, \quad (9)$$

with  $\Theta_W$  the weak mixing angle and the unitary  $2 \times 2$  mixing matrices  $U$  and  $V$  which diagonalize the chargino mass matrix, Eq. (2),  $U^* \mathcal{M}_C V^{-1} = \text{diag}(m_{\chi_1}, m_{\chi_2})$ . In Eqs. (6) and (7),  $g$  is the SU(2) weak coupling constant,  $L_e = -1/2 + \sin^2 \Theta_W$ ,  $R_e = \sin^2 \Theta_W$ ,  $\Delta(Z) = i/(s - m_Z^2)$ ,  $\Delta(\tilde{\nu}_e) = i/(t - m_{\tilde{\nu}_e}^2)$ , with  $s = (p_{e^-} + p_{e^+})^2$ ,  $t = (p_{e^-} - p_{\chi_j^-})^2$ , and  $m_{\tilde{\nu}_e}$  ( $m_Z$ ) is the mass of the electron sneutrino ( $Z$  boson). The dependence on the beam polarizations is given by the factors

$$\begin{aligned} c_{+-} &= (1 + \mathcal{P}_-)(1 - \mathcal{P}_+) , \\ c_{-+} &= (1 - \mathcal{P}_-)(1 + \mathcal{P}_+) , \end{aligned} \quad (10)$$

where  $\mathcal{P}_-$  and  $\mathcal{P}_+$  are the degrees of longitudinal polarization of the electron and positron beam, respectively, with  $-1 \leq \mathcal{P}_\pm \leq 1$ . The kinematical dependence of the spin-spin correlation terms, Eqs. (6) and (7), is given by the function [15]

$$\begin{aligned} f^{ab} &= \varepsilon_{\mu\nu\rho\sigma} \left[ s_{\chi_j^-}^{b,\mu} s_{\chi_i^+}^{a,\nu} p_{\chi_i^+}^\rho p_{\chi_j^-}^\sigma (p_{e^+} \cdot p_{\chi_j^-}) - s_{\chi_j^-}^{b,\mu} s_{\chi_i^+}^{a,\nu} p_{e^+}^\rho p_{\chi_j^-}^\sigma (p_{\chi_i^+} \cdot p_{e^-}) \right. \\ &\quad \left. + s_{\chi_i^+}^{a,\mu} p_{\chi_i^+}^\nu p_{e^-}^\rho p_{\chi_j^-}^\sigma (p_{e^+} \cdot s_{\chi_j^-}^b) + s_{\chi_j^-}^{b,\mu} p_{\chi_i^+}^\nu p_{e^+}^\rho p_{\chi_j^-}^\sigma (p_{e^-} \cdot s_{\chi_i^+}^a) \right], \end{aligned} \quad (11)$$

with  $\varepsilon_{0123} = -1$ .

Note that the spin-spin correlation terms in Eqs. (6) and (7) depend on the imaginary parts of the products of chargino couplings,  $\text{Im}(O_{ij}^{'L} O_{ij}^{'R*})$  and  $\text{Im}(V_{i1}^* V_{j1} O_{ij}^{'R})$ , and thus are manifestly CP-sensitive, i.e., sensitive to the phase  $\phi_\mu$  of the chargino sector. We also give the spin-spin correlation terms in the laboratory system in Appendix C.

### 3 Identifying the T-odd products in the spin-spin correlation terms

For an identification of the T-odd products in chargino production and decay, we consider the kinematical dependence of the spin-spin correlation terms of the amplitude squared, Eq. (5),

$$\begin{aligned}
& \sum_{a,b=1}^3 \Sigma_P^{ab} \Sigma_{D_i}^a \Sigma_{D_j}^b \propto \sum_{a,b=1}^3 f^{ab} \cdot (s_{\chi_i^+}^a \cdot p_{\ell^+}) \cdot (s_{\chi_j^-}^b \cdot p_{\ell^-}) = \\
& \epsilon_{\mu\nu\rho\sigma} \left[ p_{\ell^+}^\mu p_{\ell^+}^\nu p_{\chi_i^+}^\rho p_{e^-}^\sigma (p_{e^+} \cdot p_{\chi_j^-}) + p_{\ell^+}^\mu p_{\ell^+}^\nu p_{\chi_j^-}^\rho p_{e^+}^\sigma (p_{e^-} \cdot p_{\chi_i^+}) \right. \\
& \left. + p_{\ell^+}^\mu p_{\chi_i^+}^\nu p_{e^-}^\rho p_{\chi_j^-}^\sigma (p_{e^+} \cdot p_{\ell^-}) + p_{\ell^+}^\mu p_{\chi_i^+}^\nu p_{e^+}^\rho p_{\chi_j^-}^\sigma (p_{e^-} \cdot p_{\ell^+}) \right] , \tag{12}
\end{aligned}$$

where the scalar products  $(s_{\chi_i^+}^a \cdot p_{\ell^+})$  and  $(s_{\chi_j^-}^b \cdot p_{\ell^-})$  appear in  $\Sigma_{D_i}^a$  and  $\Sigma_{D_j}^b$ , respectively, see Eq. (A.2). We have used the explicit expression for  $f^{ab}$ , Eq. (11), and the completeness relation for the chargino spin vectors [15,19]

$$\sum_c s_{\chi_k^\pm}^{c,\mu} \cdot s_{\chi_k^\pm}^{c,\nu} = -g^{\mu\nu} + \frac{p_{\chi_k^\pm}^\mu p_{\chi_k^\pm}^\nu}{m_{\chi_k}^2} . \tag{13}$$

If we now substitute in the center-of-mass system the chargino 3-momenta by the corresponding lepton 3-momenta  $\vec{p}_{\chi_i^+} \rightarrow \vec{p}_{\ell^+}$ ,  $\vec{p}_{\chi_j^-} \rightarrow \vec{p}_{\ell^-}$ , on the right hand side of Eq. (12), we find the T-odd product

$$\mathcal{O}_T = \hat{p}_{e^-} \cdot (\hat{p}_{\ell^+} + \hat{p}_{\ell^-}) \hat{p}_{e^-} \cdot (\hat{p}_{\ell^+} \times \hat{p}_{\ell^-}) , \tag{14}$$

of the unit momentum vectors  $\hat{p} = \vec{p}/|\vec{p}|$ . If we do not replace the chargino momenta, we find an additional T-odd product

$$\mathcal{O}_T^{\text{prod}} = (\hat{p}_{e^-} \cdot \hat{p}_{\ell^-}) \hat{p}_{e^-} \cdot (\hat{p}_{\chi_j^-} \times \hat{p}_{\ell^+}) + (\hat{p}_{e^-} \cdot \hat{p}_{\ell^+}) \hat{p}_{e^-} \cdot (\hat{p}_{\chi_j^-} \times \hat{p}_{\ell^-}) . \tag{15}$$

Since the T-odd product  $\mathcal{O}_T^{\text{prod}}$  includes the chargino momentum  $\hat{p}_{\chi_j^-}$ , it will require the reconstruction of the production plane. However, as we will see later, this product gives rise to larger CP-sensitive observables. Note that in writing all momentum vectors as unit vectors, the T-odd products  $\mathcal{O}_T$  and  $\mathcal{O}_T^{\text{prod}}$  are dimensionless.

## 4 CP-sensitive observables

In this Section, we define our CP-sensitive observables, which depend on the T-odd parts of the spin-spin correlations for chargino production and decay. For an operator  $\hat{\mathcal{O}}$ , we define its expectation value by

$$\langle \hat{\mathcal{O}} \rangle = \frac{\int \hat{\mathcal{O}} |T|^2 \text{dLips}}{\int |T|^2 \text{dLips}} = \frac{1}{\sigma} \int \hat{\mathcal{O}} \frac{\text{d}\sigma}{\text{dLips}} \text{dLips} . \quad (16)$$

We now define two classes of CP observables; one class requires the reconstruction of the chargino momenta, the other class not.

### 4.1 CP-sensitive observables without the knowledge of the production plane

Using Eq. (16) and the T-odd product  $\mathcal{O}_T$ , Eq. (14), we define the two CP-sensitive observables

$$\langle \mathcal{O}_T \rangle \quad \text{and} \quad \langle \text{Sgn}(\mathcal{O}_T) \rangle . \quad (17)$$

The integration in Eq. (16) with  $\mathcal{O}_T$  and  $\text{Sgn}(\mathcal{O}_T)$  as operators, projects out the CP-sensitive parts in the spin-spin correlation terms of the amplitude squared. The contributions from the terms of the spin correlations between production and decay in Eq. (5),  $\Sigma_P^a \Sigma_{D_i}^a D_j$  and  $\Sigma_P^b \Sigma_{D_j}^b D_i$ , cancel each other. The observable  $\langle \text{Sgn}(\mathcal{O}_T) \rangle$  represents an up-down asymmetry, which gives the relative number of events for which the sign of the T-odd product  $\mathcal{O}_T$  in Eq. (14) is positive ( $N_+/N$ ), subtracted from the relative number of events where it is negative ( $N_-/N$ ).  $\langle \mathcal{O}_T \rangle$ , on the other hand, gives the expectation value of the momentum configuration  $\mathcal{O}_T$ , Eq. (14), for the event sample. Note that, since  $\mathcal{O}_T$  does not include the chargino momenta, the CP-sensitive observables in Eq. (17) can be probed without the knowledge of the production plane. Note further, that for the production of the charge conjugated pair of charginos,  $e^+e^- \rightarrow \tilde{\chi}_i^- \tilde{\chi}_j^+ \rightarrow \tilde{\nu}_\ell \ell^- \tilde{\nu}_{\ell'} \ell'^+$ , the observables  $\langle \mathcal{O}_T \rangle$  and  $\langle \text{Sgn}(\mathcal{O}_T) \rangle$  change sign. In order that the two observables from the two chargino production processes  $e^+e^- \rightarrow \tilde{\chi}_1^- \tilde{\chi}_2^+$  and  $e^+e^- \rightarrow \tilde{\chi}_2^- \tilde{\chi}_1^+$ , do not sum up to zero, one has to distinguish from which chargino ( $\tilde{\chi}_1^\pm$  or  $\tilde{\chi}_2^\mp$ ) the final state leptons originate. This can be achieved by using the different energy distributions of the leptons, since their kinematical limits depend on the mass of the decaying chargino. In Subsection 4.4, we give a numerical example showing how the leptons can be distinguished by using their different energy distributions.

#### 4.2 CP-sensitive observables which require the knowledge of the production plane

For the T-odd product  $\mathcal{O}_T^{\text{prod}}$ , Eq. (15), we analogously obtain the CP observables

$$\langle \mathcal{O}_T^{\text{prod}} \rangle \quad \text{and} \quad \langle \text{Sgn}(\mathcal{O}_T^{\text{prod}}) \rangle. \quad (18)$$

The CP-sensitive observables in Eq. (18) not only receive contributions from the spin-spin correlation terms, but also from the spin correlation terms between production and decay,  $\Sigma_P^a \Sigma_{D_i}^a D_j$  and  $\Sigma_P^b \Sigma_{D_j}^b D_i$ , see Eq. (5).

Since the T-odd product  $\mathcal{O}_T^{\text{prod}}$  includes the chargino momenta, a measurement of the CP-sensitive observables requires the reconstruction of the production plane. The extent to which such a reconstruction can be accomplished depends on the decay pattern of the produced charginos. For their subsequent two-body decays which we consider here,  $e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp \rightarrow \tilde{\nu}_\ell \ell^+ \tilde{\nu}_{\ell'} \ell'^-$ , the chargino momentum three-vector can be reconstructed up to a sign ambiguity in its second component, if the masses of the involved particles are known [20–22]. The two solutions (true and false) can then be combined using statistical methods, as shown in [21].

#### 4.3 Theoretical statistical significances

We have defined two kinds of CP-sensitive observables,  $\langle \mathcal{O} \rangle$  and  $\langle \text{Sgn}(\mathcal{O}) \rangle$ , based on the T-odd products  $\mathcal{O} = \mathcal{O}_T, \mathcal{O}_T^{\text{prod}}$ . The observable  $\langle \mathcal{O} \rangle$  is obtained by matching the complete kinematical (angular) dependence of the spin-spin correlation terms in the amplitude squared. In the literature, this technique is known by the name *optimal observables* [22,23]. In order to compare the two kinds of observables, we define their theoretical statistical significances. A comparison of the numerical values of  $\langle \mathcal{O} \rangle$  and  $\langle \text{Sgn}(\mathcal{O}) \rangle$  alone cannot be used to decide which observable is more sensitive to the CP phases. The statistical significances also allow us to compare the observables which are based on the T-odd product  $\mathcal{O}_T$ , with those which are based on  $\mathcal{O}_T^{\text{prod}}$ , which includes the chargino momentum.

The theoretical statistical significance of the CP observable  $\langle \hat{\mathcal{O}} \rangle$ , where  $\hat{\mathcal{O}} = \mathcal{O}_T, \mathcal{O}_T^{\text{prod}}$  or  $\hat{\mathcal{O}} = \text{Sgn}(\mathcal{O}_T), \text{Sgn}(\mathcal{O}_T^{\text{prod}})$ , is defined by [22,24]

$$S[\hat{\mathcal{O}}] = \sqrt{N} \frac{|\langle \hat{\mathcal{O}} \rangle|}{\sqrt{\langle \hat{\mathcal{O}}^2 \rangle}}, \quad (19)$$

with the number of events  $N = 8 \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-) \times \text{BR}(\tilde{\chi}_1^+ \rightarrow \tilde{\nu}_e e^+) \times \text{BR}(\tilde{\chi}_2^- \rightarrow \tilde{\nu}_e e^-) \times \mathcal{L}$ , where  $\mathcal{L}$  denotes the integrated luminosity. The factor 8 appears since there are 4 possibilities to sum over the lepton flavors  $\ell = e, \mu$ , and two charge



assignments for chargino production  $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp)$ . The quantity  $S[\hat{\mathcal{O}}]/\sqrt{\mathcal{L}}$  is called *effective asymmetry* in [22].

We have obtained the significance  $S[\hat{\mathcal{O}}]$ , Eq. (19), by imposing that the observable should at least be larger than its absolute statistical error [24]

$$\frac{|\langle \hat{\mathcal{O}} \rangle|}{\Delta \langle \hat{\mathcal{O}} \rangle} > 1, \quad (20)$$

where the absolute error is approximated by

$$\Delta \langle \hat{\mathcal{O}} \rangle = \frac{S[\hat{\mathcal{O}}]}{\sqrt{N}} \sqrt{\langle \hat{\mathcal{O}}^2 \rangle - \langle \hat{\mathcal{O}} \rangle^2} \simeq \frac{S[\hat{\mathcal{O}}]}{\sqrt{N}} \sqrt{\langle \hat{\mathcal{O}}^2 \rangle}. \quad (21)$$

The theoretical statistical significance is thus equal to the number of standard deviations to which the corresponding CP observable can be determined to be non-zero. For an ideal detector, a significance of, e.g.,  $S = 1$  implies that the CP observables can be measured at the statistical 68% confidence level. We remark that the theoretical statistical significance in Eq. (19) is solely a theoretical definition. Background and detector simulations for particle reconstruction efficiencies are not included. In order to give realistic values of the statistical significances, a detailed Monte Carlo analysis would be required, which is however beyond the scope of the present work. In the following we comment on the major SUSY and SM backgrounds and discuss how they can be reduced.

#### 4.4 Lepton energy distributions and backgrounds

A main SM background will be from  $W$  pair production,  $e^+e^- \rightarrow W^+W^-$ , where both  $W$ 's decay leptonically  $W^+ \rightarrow \nu_\ell \ell^+$  and  $W^- \rightarrow \bar{\nu}_\ell \ell^-$ . For  $\sqrt{s} = 500$  GeV and unpolarized beams the cross section is  $\sigma(e^+e^- \rightarrow W^+W^-) = 7.4$  pb [25], and  $\sum_{\ell, \ell'} \text{BR}(W^+ \rightarrow \nu_\ell \ell^+) \text{BR}(W^- \rightarrow \bar{\nu}_{\ell'} \ell'^-) = 4.4\%$ , for  $\ell, \ell' = e, \mu$  [26]. With a beam polarization of  $(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, 0.6)$ , the cross section for  $e^+e^- \rightarrow W^+W^-$  is about a factor 3 larger [25]. A main SUSY background would be from the pair production of equal charginos,  $e^+e^- \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^+$  and  $e^+e^- \rightarrow \tilde{\chi}_2^- \tilde{\chi}_2^+$ . For the scenario as given in Table 1, the cross sections are  $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^+) = 974$  fb, and  $\sigma(e^+e^- \rightarrow \tilde{\chi}_2^- \tilde{\chi}_2^+) = 145$  fb at  $\sqrt{s} = 500$  GeV with  $(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, 0.6)$ . The signal cross section is  $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-) = 527$  fb. The chargino branching ratios are  $\text{BR}(\tilde{\chi}_1^+ \rightarrow \tilde{\nu}_e e^+) = 33\%$  and  $\text{BR}(\tilde{\chi}_2^+ \rightarrow \tilde{\nu}_e e^+) = 8\%$ .

A large part of the background can be cut by using the different energy distributions of the final state leptons. In Fig. 2, we show the normalized energy distributions of the leptons  $\ell^-$  stemming from the various reactions  $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ ;  $\tilde{\chi}_j^- \rightarrow \tilde{\nu}_\ell \ell^-$ , where we have neglected the effect of the longitudinal chargino polarization which

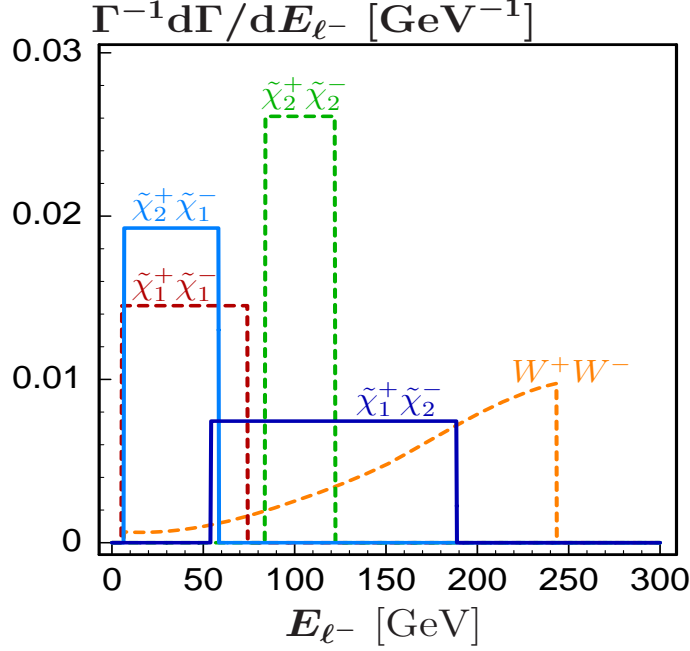


Fig. 2. Normalized energy distributions of the signal leptons  $\ell^-$  from chargino pair production  $e^+e^- \rightarrow \tilde{\chi}_2^+\tilde{\chi}_1^-$ ;  $\tilde{\chi}_1^- \rightarrow \tilde{\nu}_\ell \ell^-$  (light blue line) and  $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^-$ ;  $\tilde{\chi}_2^- \rightarrow \tilde{\nu}_\ell \ell^-$  (dark blue line), for the parameters as given in Table 1. Furthermore, we show the energy distributions of the background leptons  $\ell^-$  from the reactions  $e^+e^- \rightarrow W^+W^-$ ;  $W^- \rightarrow \tilde{\nu}_\ell \ell^-$  (dashed orange line [27]),  $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-$ ;  $\tilde{\chi}_1^- \rightarrow \tilde{\nu}_\ell \ell^-$  (dashed red line), and  $e^+e^- \rightarrow \tilde{\chi}_2^+\tilde{\chi}_2^-$ ;  $\tilde{\chi}_2^- \rightarrow \tilde{\nu}_\ell \ell^-$  (dashed green line).

may somewhat change the box-shape of the distributions. For the energy distribution of the leptons  $\ell^-$  from  $e^+e^- \rightarrow W^+W^-$ ;  $W^- \rightarrow \tilde{\nu}_\ell \ell^-$ , we have fitted the curve in Fig. 3 of Ref. [27].

We can observe from Fig. 2 that the energy distribution of the signal leptons  $\ell^-$  from  $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^-$ ,  $\tilde{\chi}_1^- \rightarrow \tilde{\nu}_\ell \ell^-$  (light blue line), does only slightly overlap with that from the charge conjugated process  $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^-$ ,  $\tilde{\chi}_2^- \rightarrow \tilde{\nu}_\ell \ell^-$  (dark blue line) in this scenario. In the first place, this is essential for a measurement of the CP observables, since they change sign for the charge conjugated process. Secondly, the background from equal charginos pair production  $e^+e^- \rightarrow \tilde{\chi}_2^+\tilde{\chi}_2^-$  can be totally eliminated, since both leptons have an energy in the interval  $E_\ell \in [84, 122]$  GeV. As discussed above, a signal event has a lepton with energy  $E_\ell \in [7, 59]$  GeV, and an oppositely charged lepton with energy  $E_\ell \in [54, 189]$  GeV. On the other hand, the lepton energy distribution from  $\tilde{\chi}_1^+\tilde{\chi}_1^-$  production overlaps with that of a signal lepton (the one from the  $\tilde{\chi}_2^\pm$  decay) only in the interval  $E_\ell \in [54, 74]$  GeV. The background from  $\tilde{\chi}_1^+\tilde{\chi}_1^-$  production can be eliminated, if we require one lepton with  $E_\ell > 74$  GeV.

The energy distribution of the leptons from  $W^\pm$  decays overlaps with that from the signal in the whole range, see Fig. 2. However, the signal involves one lepton with a rather low energy  $E_\ell \in [7, 59]$  GeV, where the background is low. In this interval, the signal is about twice as large as the  $W^\pm$  background. Thus, as any CP-even background, which contributes only to the denominator but not to the numerator

of the CP observables, it will reduce them and the corresponding significances only slightly. Moreover, the high energetic background leptons from  $W^\pm$  decays can be significantly reduced by the cut  $E_\ell \lesssim 189$  GeV, see Fig. 2.

Additional important SUSY background originates from the following processes [28,29]:

(a) stau pair production  $e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$ , with subsequent decays  $\tilde{\tau} \rightarrow \tau\tilde{\chi}_1^0$ , followed by leptonic tau decays, (b) selectron and smuon production  $e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^-$ ,  $\ell = e, \mu$ , followed by  $\tilde{\ell} \rightarrow \ell\tilde{\chi}_1^0$ , and (c) neutralino production  $e^+e^- \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0$  and their decays into leptons via sleptons.

Additional SM background reactions are [28,29]: (i) photon-induced tau pair production  $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ , followed by leptonic tau decays, (ii) photon-induced  $W$  pair production  $e^+e^- \rightarrow e^+e^-W^+W^-$ , with  $W^\pm \rightarrow \ell^\pm\nu_\ell$ , (iii) tau pair production followed by leptonic tau decays, (iv)  $Z$  pair production followed by the decays into leptons, and (v) single boson production,  $e^+e^-Z$  and  $\nu\bar{\nu}Z$  with  $Z \rightarrow \ell^+\ell^-$ ,  $\nu\bar{\nu}$ .

Recently a detailed NLO study of chargino pair production  $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-$  and decay  $\tilde{\chi}_1^\pm \rightarrow \ell^\pm\tilde{\nu}_\ell$  at a linear collider with  $\sqrt{s} = 1$  TeV has been performed [28]. The SM and SUSY backgrounds to the experimental signature  $e^\pm\mu^\mp + \cancel{E}$  have been taken into account. A signal to background ratio of 0.62 has been obtained after appropriate cuts [28]. In particular, the background from photon-induced  $\tau$  pair production, which may exceed the size of the signal cross section by a factor of  $10^4$ , has been reduced by a factor of  $10^6$ . Further the authors of Ref. [28] have shown that SM processes lead to a flat background distribution, which can be easily subtracted, while SUSY backgrounds are more challenging, since their kinematic distributions are similar to the signal in general. We expect that our proposed discrimination criteria for the lepton energies, together with cuts as applied in [28,29], will enhance the signal to background ratio also for our observables.

## 5 Numerical results

We present numerical results for the CP-sensitive observables defined in Section 4 for chargino production and decay,  $e^+e^- \rightarrow \tilde{\chi}_1^\pm\tilde{\chi}_2^\mp \rightarrow \tilde{\nu}_\ell\ell^+\tilde{\nu}_{\ell'}\ell'^-$ , for  $\ell, \ell' = e, \mu$ . We study the dependence of the CP observables on the phase of the higgsino mass parameter  $\mu = |\mu|e^{i\phi_\mu}$  in the framework of the general MSSM, where restrictions on  $\phi_\mu$  from the electron and neutron EDMs are less severe compared to the constrained MSSM [4,5]. For example, the cancellation of various contributions to the EDMs allow for the possibility of  $\phi_\mu \sim \mathcal{O}(1)$  [5]. Since our analysis does not include all relevant parameters necessary to predict the actual values of the EDMs, we do not take the EDMs into account, and show the full  $\phi_\mu$ -dependence of the observables.

Our study is for the ILC with  $\sqrt{s} = 500$  GeV and longitudinal beam polarizations  $(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, 0.6)$ . This choice enhances the  $\tilde{\nu}_e$  exchange contribution, yielding

larger cross sections and CP observables. We also provide numerical results for the chargino cross sections and the branching ratios. Furthermore, we give the theoretical statistical significances to which the CP-sensitive observables can be determined to be non-zero.

For the calculation of the chargino decay widths and branching ratios we consider their two-body decays [30]

$$\begin{aligned} \tilde{\chi}_{1,2}^{\pm} &\rightarrow W^{\pm} \tilde{\chi}_n^0, e^{\pm} \tilde{\nu}_e, \mu^{\pm} \tilde{\nu}_{\mu}, \tau^{\pm} \tilde{\nu}_{\tau}, \nu_e \tilde{e}_L^{\pm}, \nu_{\mu} \tilde{\mu}_L^{\pm}, \nu_{\tau} \tilde{\tau}_{1,2}^{\pm}, \\ \tilde{\chi}_2^{\pm} &\rightarrow Z \tilde{\chi}_1^{\pm}, h \tilde{\chi}_1^{\pm}. \end{aligned} \quad (22)$$

We assume the GUT inspired relation  $|M_1| = 5/3 M_2 \tan^2 \Theta_W$ , and in the stau sector we fix the trilinear scalar coupling parameter  $A_{\tau} = 250$  GeV.

$M_2$	$ \mu $	$\phi_{\mu}$	$\tan \beta$	$m_{\tilde{\nu}_{e,\mu}}$	$m_{\chi_1}$	$m_{\chi_2}$
152	200	$0.5\pi$	3	103	125	246

Table 1

Input parameters  $M_2$ ,  $|\mu|$ ,  $\phi_{\mu}$ ,  $\tan \beta$ , and  $m_{\tilde{\nu}_{e,\mu}}$ . All mass parameters are given in GeV. The other masses are  $m_{\tilde{e}_R} = 107$  GeV,  $m_{\tilde{e}_L} = 125$  GeV,  $m_{\tilde{\tau}_1} = 106$  GeV,  $m_{\tilde{\tau}_2} = 127$  GeV,  $m_{\chi_1^0} = 73$  GeV,  $m_{\chi_2^0} = 127$  GeV,  $m_{\chi_3^0} = 208$  GeV,  $m_{\chi_4^0} = 247$  GeV.

Before resuming with the numerical investigation, we address the parameter dependence of the CP-sensitive coupling factors  $\text{Im}(O'_{12}{}^L O'_{12}{}^{R*})$  and  $\text{Im}(V_{11}^* V_{21} O'_{12}{}^R)$  on which our CP-sensitive observables depend, see Eqs. (6) and (7). When we expand them by using the parametrization of the chargino mixing matrices  $U$  and  $V$ , we find

$$\text{Im}(V_{11}^* V_{21} O'_{12}{}^R) = 2 \text{Im}(O'_{12}{}^L O'_{12}{}^{R*}) = \frac{1}{8} \sin 2\theta_1 \sin 2\theta_2 \sin(\phi_1 - \phi_2 + \gamma_1 - \gamma_2), \quad (23)$$

with the chargino mixing angles  $\theta_1, \theta_2$ , and the phases  $\phi_1, \gamma_1, \phi_2, \gamma_2$  of the matrices  $U, V$ . Their explicit dependence on the parameters of the chargino system can be found in [7,16]. In particular, the phases are zero when  $\phi_{\mu}$  is zero. One finds that the CP-sensitive coupling factors (and therefore the CP observables) are largest for large gaugino-higgsino mixing, i.e. for  $M_2 \sim |\mu|$ . Furthermore, a small value for  $\tan \beta$  is preferable, as  $\tan \beta \rightarrow \infty$  results in  $\phi_1, \gamma_2 \rightarrow \phi_{\mu}$  and  $\phi_2, \gamma_1 \rightarrow 0$ , leading to  $\sin(\phi_1 - \phi_2 + \gamma_1 - \gamma_2) \rightarrow 0$ . We therefore choose a mixed scenario with small  $\tan \beta$ , see the parameters in Table 1.

In Fig. 3a, we show the  $\phi_{\mu}$ -dependence of the CP-sensitive observables  $\langle \mathcal{O}_T \rangle$  and  $\langle \mathcal{O}_T^{\text{prod}} \rangle$ , Eq. (17) and (18), respectively. One can clearly see their asymmetric dependence on  $\phi_{\mu}$ . At  $\phi_{\mu} = 0 \pmod{\pi}$ , the chargino couplings are real and therefore

the CP-sensitive observables vanish. The observables attain their largest values at  $\phi_\mu = 0.75\pi$  of about  $\langle \mathcal{O}_T \rangle = -6.5 \cdot 10^{-3}$  and  $\langle \mathcal{O}_T^{\text{prod}} \rangle = 2 \cdot 10^{-2}$ . Fig. 3b shows the  $\phi_\mu$ -dependence of the CP-sensitive observables  $\langle \text{Sgn}(\mathcal{O}_T) \rangle$  and  $\langle \text{Sgn}(\mathcal{O}_T^{\text{prod}}) \rangle$ . At  $\phi_\mu = 0.75\pi$  the observables reach  $\langle \text{Sgn}(\mathcal{O}_T) \rangle = -1.9\%$  and  $\langle \text{Sgn}(\mathcal{O}_T^{\text{prod}}) \rangle = 5.6\%$ .

In Fig. 4a, we show the chargino production cross section  $\sigma_{12} \equiv \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-)$  as a function of  $\phi_\mu$ . The production cross section varies between  $\sigma_{12} = 618$  fb for  $\phi_\mu = 0 \pmod{2\pi}$ , and  $\sigma_{12} = 227$  fb for  $\phi_\mu = \pi$ . Fig. 4b shows the branching ratios  $\text{BR}(\tilde{\chi}_1^+ \rightarrow \tilde{\nu}_e e^+)$  and  $\text{BR}(\tilde{\chi}_2^- \rightarrow \tilde{\nu}_e e^-)$ , which are about 32% and 7%, respectively. The  $\phi_\mu$ -dependence of the cross section  $\sigma \equiv \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^- \rightarrow \tilde{\nu}_e e^+ \tilde{\nu}_e e^-)$  is given in Fig. 4c. In contrast to the CP-sensitive observables, the cross section shows a symmetric dependence on  $\phi_\mu$ . The phase ambiguity  $\phi_\mu \leftrightarrow 2\pi - \phi_\mu$  of the higgsino mass parameter  $\mu$  can only be resolved by a measurement of CP-sensitive observables. In Fig. 4d, we present the theoretical statistical significances,  $S[\hat{\mathcal{O}}]$ , as defined in Eq. (19), for an integrated luminosity  $\mathcal{L} = 500 \text{ fb}^{-1}$ . For  $\langle \text{Sgn}(\mathcal{O}_T) \rangle$  and  $\langle \mathcal{O}_T \rangle$ , which do not require a reconstruction of the production plane, the theoretical statistical significances reach 4 and 5 standard deviations, respectively. The theoretical statistical significances of the CP-sensitive observables  $\langle \text{Sgn}(\mathcal{O}_T^{\text{prod}}) \rangle$  and  $\langle \mathcal{O}_T^{\text{prod}} \rangle$  are at the 12- $\sigma$  level for the considered scenario.

We note that also other leptonic chargino decay chains, i.e.  $\tilde{\chi}_{1,2}^\pm \rightarrow \tilde{\ell}^\pm \nu$ ;  $\tilde{\ell}^\pm \rightarrow \tilde{\chi}_1^0 \ell^\pm$  and  $\tilde{\chi}_{1,2}^\pm \rightarrow W^\pm \tilde{\chi}_1^0$ ;  $W^\pm \rightarrow \nu \ell^\pm$ , can be used for measuring the spin-spin correlation terms of the chargino production amplitude. The inclusion of these decay chains can be done in a similar fashion as in Ref. [18]. However, we expect that this would not lead to larger statistical significances of the CP-sensitive observables. Although the total number of events is increased, the CP-sensitive observables are reduced, since they have to be weighted with the corresponding decay branching ratios [18].

Finally, note that the T-odd products, Eqs. (14) and (15), can also be used for the definition of CP-sensitive observables in neutralino production with subsequent decays. This is possible for the pair production  $\sigma(e^+e^- \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)$ , with  $i > j \geq 2$ . The neutralino polarizations can then again be obtained from the lepton distributions, e.g., in the decays  $\tilde{\chi}_{i,j}^0 \rightarrow \tilde{\ell}^\pm \ell^\mp$  [31].

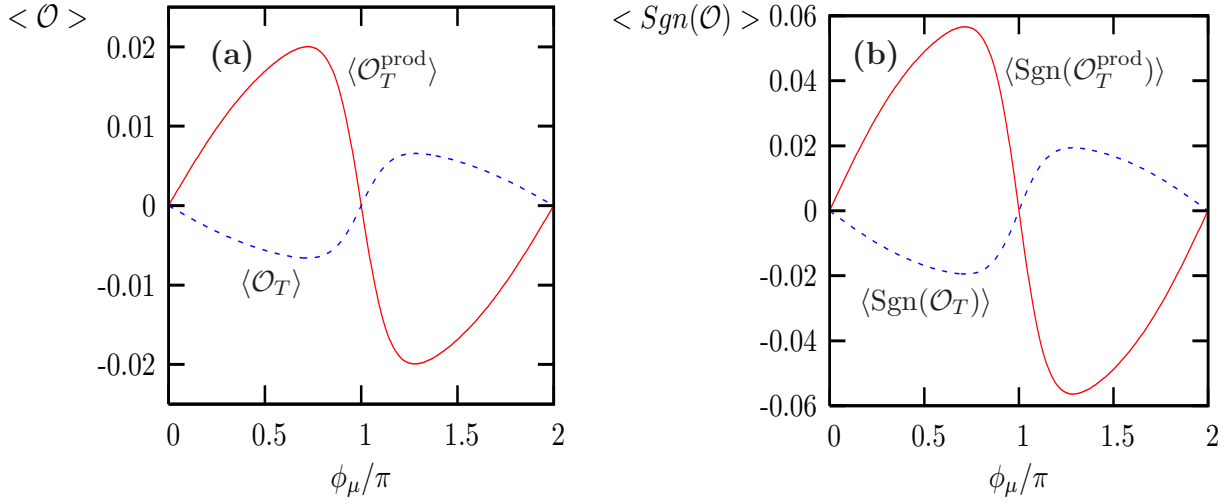


Fig. 3.  $\phi_\mu$ -dependence of the CP-sensitive observables (a)  $\langle \mathcal{O}_T^{\text{prod}} \rangle$  (red solid line) and  $\langle \mathcal{O}_T \rangle$  (blue dashed line), and (b)  $\langle \text{Sgn}(\mathcal{O}_T^{\text{prod}}) \rangle$  (red solid line) and  $\langle \text{Sgn}(\mathcal{O}_T) \rangle$  (blue dashed line), for chargino production and decay  $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^- \rightarrow \tilde{\nu}_e e^+ \tilde{\nu}_e e^-$ , for the scenario defined in Table 1, at  $\sqrt{s} = 500$  GeV with longitudinal beam polarizations  $(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, 0.6)$ .

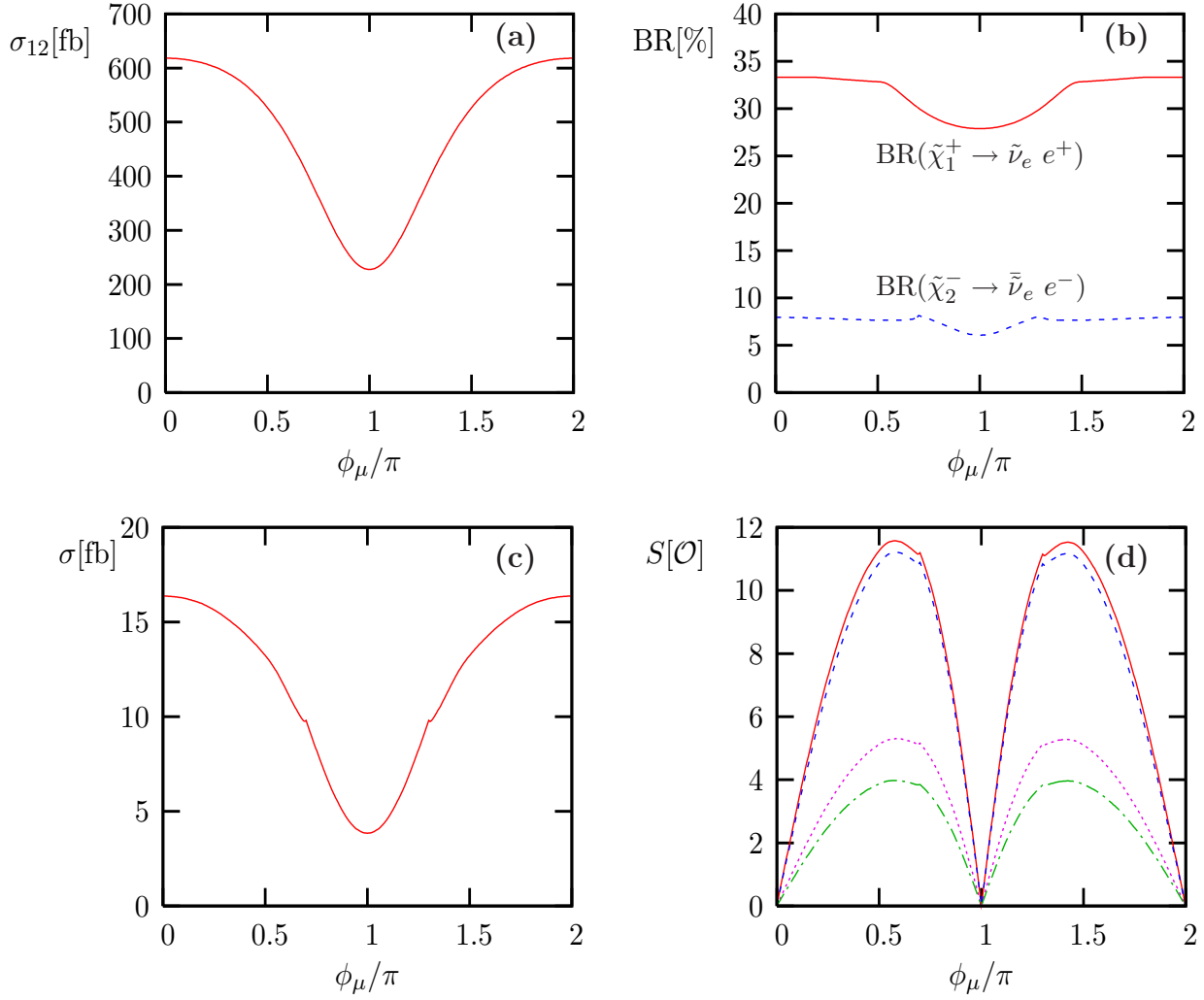


Fig. 4.  $\phi_\mu$ -dependence of (a) the chargino production cross section  $\sigma_{12} \equiv \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-)$ , (b) the branching ratios  $\text{BR}(\tilde{\chi}_1^+ \rightarrow \tilde{\nu}_e e^+)$  (red solid line) and  $\text{BR}(\tilde{\chi}_2^- \rightarrow \tilde{\nu}_e e^-)$  (blue dashed line), (c) the cross section  $\sigma \equiv \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^- \rightarrow \tilde{\nu}_e e^+ \tilde{\nu}_e e^-)$ , and (d) the theoretical statistical significances  $S[\text{Sgn}(\mathcal{O}_T^{\text{prod}})]$  (red solid line),  $S[\mathcal{O}_T^{\text{prod}}]$  (blue dashed line),  $S[\mathcal{O}_T]$  (magenta dotted line), and  $S[\text{Sgn}(\mathcal{O}_T)]$  (green dot-dashed line), for the scenario defined in Table 1, at  $\sqrt{s} = 500$  GeV with longitudinal beam polarizations  $(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, 0.6)$ , and for (d), with an integrated luminosity  $\mathcal{L} = 500 \text{ fb}^{-1}$ .

## 6 Summary and conclusions

We have proposed novel CP-sensitive observables in chargino production  $e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ . These CP observables are sensitive to the phase of the higgsino parameter  $\mu$ . They arise on tree-level, and rely on T-odd products in the chargino spin-spin correlations. These are the terms of the matrix element, which include the polarizations of both charginos, with one component perpendicular to the production plane. The chargino polarization can be deduced from the distributions of their leptonic decay products  $\tilde{\chi}_{1,2}^\pm \rightarrow \tilde{\nu}_\ell \ell^\pm$ ,  $\ell = e, \mu$ .

In order to probe the CP-sensitive spin-spin correlation terms, we have identified two different T-odd products. The first one,  $\mathcal{O}_T$ , does not involve the chargino momentum, which has the advantage that it is not necessary to reconstruct the production plane. We recall that other T-odd products proposed in the literature always require such a reconstruction, if only one leptonic chargino decay is considered. The second T-odd product,  $\mathcal{O}_T^{\text{prod}}$ , in contrast includes the chargino momentum. Based on these T-odd products, we have defined two sorts of CP-sensitive observables. One is an up-down asymmetry, giving the difference of events with positive and negative T-odd products. The other sort of CP-sensitive observables are the expectation values of the T-odd products for the event sample.

In the numerical study, we have found that the observables are largest in mixed scenarios with small  $\tan\beta$ . We have defined theoretical significances to decide, which CP observable is most sensitive to the CP phase  $\phi_\mu$ . For a linear collider with  $\sqrt{s} = 500$  GeV and longitudinally polarized beams,  $(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, 0.6)$ , with an integrated luminosity of  $\mathcal{L} = 500 \text{ fb}^{-1}$ , the CP-sensitive observables that are based on the T-odd product  $\mathcal{O}_T$  yield  $S[\mathcal{O}_T] \lesssim 5$ . We find larger significances  $S[\mathcal{O}_T^{\text{prod}}] \lesssim 12$  for the CP-sensitive observables that are based on  $\mathcal{O}_T^{\text{prod}}$ . Thus the largest CP-violating effects are obtained if the chargino production plane can be reconstructed. However, only a detailed experimental study with background and detector simulations can show whether the CP-sensitive observables are accessible.

Finally, we remark that our proposed method for analyzing T-odd products in the spin-spin correlations terms can also be used for the definition of CP-sensitive observables in other fermion pair production processes, such as neutralino productions.

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## Appendix

### A Quantities $D$ and $\Sigma_D^c$ in the spin density matrix formalism

The coefficients in Eq. (5) of the chargino decay matrices for  $\tilde{\chi}_k^+ \rightarrow \tilde{\nu}_\ell \ell^+$  and  $\tilde{\chi}_k^- \rightarrow \tilde{\nu}_\ell \ell^-$ ,  $\ell = e, \mu$ , are

$$D_k = \frac{g^2}{2} |V_{k1}|^2 (m_{\chi_k}^2 - m_{\tilde{\nu}_\ell}^2) , \quad (\text{A.1})$$

and

$$\Sigma_{D_k}^c = \pm g^2 |V_{k1}|^2 m_{\chi_k} (s_{\chi_k^\mp}^c p_{\ell^\mp}) . \quad (\text{A.2})$$

The positive sign in Eq. (A.2) holds for the decay  $\tilde{\chi}_k^- \rightarrow \tilde{\nu}_\ell \ell^-$ , and the negative sign for the charge conjugated decay  $\tilde{\chi}_k^+ \rightarrow \tilde{\nu}_\ell \ell^+$ .

### B Momentum and polarization vectors

We choose a coordinate system with the  $z$ -axis along the  $\vec{p}_{e^-}$  direction in the center-of-mass system. The 4-momenta of the charginos  $\tilde{\chi}_i^-$  and  $\tilde{\chi}_j^+$  are

$$\begin{aligned} p_{\chi_i^+} &= q(E_{\chi_i}/q, -\sin\theta, 0, -\cos\theta) , \\ p_{\chi_j^-} &= q(E_{\chi_j}/q, \sin\theta, 0, \cos\theta) , \end{aligned} \quad (\text{B.1})$$

with their energies and common momentum

$$E_{\chi_{i,j}} = \frac{s + m_{\chi_{i,j}}^2 - m_{\chi_{j,i}}^2}{2\sqrt{s}} , \quad q = \frac{\lambda^{\frac{1}{2}}(s, m_{\chi_i}^2, m_{\chi_j}^2)}{2\sqrt{s}} , \quad (\text{B.2})$$

respectively, with the scattering angle  $\theta \angle(\vec{p}_{e^-}, \vec{p}_{\chi_j^-})$ , and the kinematic function  $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$ . The azimuthal angle can be set to zero, due to rotational invariance around the beam axis.

The three spin basis vectors of  $\tilde{\chi}_k^+$  and  $\tilde{\chi}_k^-$  are chosen to be

$$\begin{aligned} s_{\chi_k^\pm}^1 &= \left( 0, \frac{\vec{s}_{\chi_k^\pm}^2 \times \vec{s}_{\chi_k^\pm}^3}{|\vec{s}_{\chi_k^\pm}^2 \times \vec{s}_{\chi_k^\pm}^3|} \right) = \pm(0, \cos \theta, 0, -\sin \theta) , \\ s_{\chi_k^\pm}^2 &= \left( 0, \frac{\vec{p}_{e^-} \times \vec{p}_{\chi_k^\pm}}{|\vec{p}_{e^-} \times \vec{p}_{\chi_k^\pm}|} \right) = (0, 0, 1, 0) , \\ s_{\chi_k^\pm}^3 &= \frac{1}{m_{\chi_k}} \left( q, \frac{E_{\chi_k}}{q} \vec{p}_{\chi_k^\pm} \right) = \frac{E_{\chi_k}}{m_{\chi_k}} (q/E_{\chi_k}, \mp \sin \theta, 0, \mp \cos \theta) . \end{aligned} \quad (\text{B.3})$$

They fulfill the orthonormality relations  $s_{\chi_k^\pm}^c \cdot s_{\chi_k^\pm}^d = -\delta^{cd}$  and  $s_{\chi_k^\pm}^c \cdot p_{\chi_k^\pm} = 0$ . The 4-momenta of the leptons in the decays  $\tilde{\chi}_i^+ \rightarrow \tilde{\nu}_\ell \ell^+$  and  $\tilde{\chi}_j^- \rightarrow \tilde{\nu}_{\ell'} \ell'^-$  are

$$p_{\ell^+} = |\vec{p}_{\ell^+}| (1, \cos \phi_{\ell^+} \sin \theta_{\ell^+}, \sin \phi_{\ell^+} \sin \theta_{\ell^+}, \cos \theta_{\ell^+}) , \quad (\text{B.4})$$

$$p_{\ell'^-} = |\vec{p}_{\ell'^-}| (1, \cos \phi_{\ell'^-} \sin \theta_{\ell'^-}, \sin \phi_{\ell'^-} \sin \theta_{\ell'^-}, \cos \theta_{\ell'^-}) , \quad (\text{B.5})$$

respectively, with

$$|\vec{p}_{\ell^+}| = \frac{m_{\chi_i}^2 - m_{\tilde{\nu}_\ell}^2}{2(E_{\chi_i} + q \cos \vartheta_{\ell^+})} , \quad |\vec{p}_{\ell'^-}| = \frac{m_{\chi_j}^2 - m_{\tilde{\nu}_{\ell'}}^2}{2(E_{\chi_j} - q \cos \vartheta_{\ell'^-})} , \quad (\text{B.6})$$

and

$$\begin{aligned} \cos \vartheta_{\ell^+} &= \sin \theta \sin \theta_{\ell^+} \cos \phi_{\ell^+} + \cos \theta \cos \theta_{\ell^+} , \\ \cos \vartheta_{\ell'^-} &= \sin \theta \sin \theta_{\ell'^-} \cos \phi_{\ell'^-} + \cos \theta \cos \theta_{\ell'^-} . \end{aligned} \quad (\text{B.7})$$

## C Spin-spin correlation terms in the laboratory system

The spin-spin correlation terms in the laboratory system are

$$\Sigma_P^{12}(ZZ) = -\frac{g^4}{2 \cos^4 \Theta_W} |\Delta(Z)|^2 (L_e^2 c_{-+} + R_e^2 c_{+-}) \text{Im}(O_{ij}^{'L} O_{ij}^{'R*}) E_{\chi_i} s q \sin^2 \theta, \quad (\text{C.1})$$

$$\Sigma_P^{12}(Z\tilde{\nu}_e) = \frac{g^4}{4 \cos^2 \Theta_W} \Delta(Z) \Delta(\tilde{\nu}_e)^* L_e c_{-+} \text{Im}(V_{i1}^* V_{j1} O_{ij}^{'R}) E_{\chi_i} s q \sin^2 \theta , \quad (\text{C.2})$$

$$\Sigma_P^{23}(ZZ) = \frac{g^4}{4 \cos^4 \Theta_W} |\Delta(Z)|^2 (L_e^2 c_{-+} + R_e^2 c_{+-}) \text{Im}(O_{ij}^{'L} O_{ij}^{'R*}) m_{\chi_j} s q \sin 2\theta , \quad (\text{C.3})$$

$$\Sigma_P^{23}(Z\tilde{\nu}_e) = -\frac{g^4}{8\cos^2\Theta_W}\Delta(Z)\Delta(\tilde{\nu}_e)^*L_e c_{-+}\text{Im}(V_{i1}^*V_{j1}O_{ij}'^R) m_{\chi_j}sq\sin 2\theta, \quad (\text{C.4})$$

which we obtain by inserting the momenta and spin vectors in the laboratory system, Eqs. (B.1) and (B.3), into Eqs. (6) and (7). In order to obtain the terms  $\Sigma_P^{21}(ZZ)$  and  $\Sigma_P^{21}(Z\tilde{\nu}_e)$ , one has to exchange  $E_{\chi_i} \rightarrow -E_{\chi_j}$  in Eqs. (C.1) and (C.2). In order to obtain the terms  $\Sigma_P^{32}(ZZ)$  and  $\Sigma_P^{32}(Z\tilde{\nu}_e)$ , one has to exchange  $m_{\chi_j} \rightarrow -m_{\chi_i}$  in Eqs. (C.3) and (C.4).

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